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Free-carrier magnetoabsorption in quantum well wires

G B Ibragimov

Institute of Physics, National Academy of Sciences of the Azerbaijan Republic, pr. H Javid 33, Baku, Az 1143, Azerbaijan

E-mail: guseyn@physics.ab.az and guseyn_gb@mail.ru

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Abstract

The theory of free-carrier absorption is given for a quasi-one-dimensional semiconducting structure in a quantizing magnetic field for the case where the carriers are scattered by polar optical phonons and acoustic phonons and the radiation field is polarized perpendicular to the magnetic field direction. The usual resonance condition $P\omega_c = \Omega + \omega_0$, where P is an integer and ω_0 and ω_c are the optical-phonon frequency and cyclotron frequency, respectively, becomes $P\tilde{\omega} = \Omega \pm \omega_0$, with $\tilde{\omega}$ equal to $\sqrt{\omega_c^2 + \omega^2}$. The magnetic field dependence of the absorption for the transverse configuration can be explained in terms of a phonon-assisted transition between the various Landau levels of the carriers.

1. Introduction

The application of a magnetic field to a crystal changes the dimensionality of the electronic levels and leads to a redistribution of the density of states. Quantum well wires (QWW) in a magnetic field have been the subject of several investigations [1–5]. In [1] rectangular QWWs were treated in the decoupled approximation. Concerning theoretical work on magnetotransport in QWWs, we are aware of the Hall resistivity treatments of [2, 3] and of magnetophonon oscillations in [4]. In [5] the field-induced change in optical anisotropy was studied for a quasi-two-dimensional (Q2D) system subject to a periodic modulation. The effect of a magnetic field on conductance quantization in quasi-one-dimensional (Q1D) systems is reviewed in [6]. Progress in the techniques of growth on patterned substrates and of cleaved-edge overgrowth has led to QWWs with very good optical properties [7–10], thus renewing the interest for the basic properties of Q1D systems.

In this work we are interested in the effect of a magnetic field on the free carrier absorption (FCA) in semiconductor QWWs. Over the past two decades the investigation of FCA in low-dimensional systems has been very intense. Scattering-assisted absorption by free electrons

and holes in the active quantum wells (QWs) then usually determines the internal loss in optically pumped laser devices with undoped cladding. Even in electrically pumped devices, assisted FCA can dominate if the lasing mode is optically confined primarily to the active region, as in interband cascade lasers [11]. FCA in diode optical cladding layers consisting of superlattice injectors [12] can also be significant. FCA is one of the powerful means to understand the scattering mechanisms of carriers. In bulk semiconductors it accounts for the absorption of electromagnetic radiation of frequencies Ω such that $\hbar\Omega < E_g$, where E_g is the band gap [13]. In QW structures, apart from the direct interband and intersubband optical transitions, optical absorption can also take place via indirect intrassubband optical transitions in which carriers absorb or emit a photon with a simultaneous scattering from phonons or other imperfections. The quantum theory for FCA in Q2D structures is well developed both in the absence [14–24] and in the presence of quantizing magnetic fields [25]. In previous work [25] we have extended the theory of FCA in Q2D systems in the presence of a quantizing magnetic field when phonon scattering is important, and it was found that the FCA coefficient oscillates as a function of the magnetic field and photon frequency with resonances occurring when $P\omega_c = \Omega \pm \omega_0$, where ω_c, Ω and ω_0 are the cyclotron, photon and phonon frequency, respectively, and where P is an integer.

The optical properties of quantum wires are well understood theoretically [26–29]. Forshaw and Whittaker [26] have presented a method to calculate accurately excitonic spectra in QWWs, and have described the transition of an exciton confinement in two directions to confinement in just one dimension. Intraband [27] and interband [28] optical absorption in Q1D systems in magnetic fields has been studied. Glutsch and Chemla [29] have calculated the optical absorption of QWWs for a large variety of wire widths, taking into account Coulomb interaction, unequal electron and hole effective masses, and continuum states. The theory of FCA has been studied theoretically in Q1D structures only in the absence of a quantizing magnetic field [30–32].

Experimental work has been done on magneto-transport [33] and magneto-optical absorption [34–36] in QWWs. In the magneto-photoluminescence and magneto-absorption experiments [34], characteristic features of low-dimensional excitons in high magnetic fields were observed. Far-infrared spectroscopy [35] on quantum structures with tailored nonparabolic potential give a detailed insight into complex many-body effects of quantum wires. Photoluminescence and photoluminescence excitation spectroscopy [36] have been performed at liquid helium temperatures in external magnetic fields up to 7.5 T. In the cyclotron resonance experiments [34] with the magnetic field tilted away from the growth direction, new features originating from the subband-Landau-level coupling were found.

In this paper we extend the quantum theory of the FCA developed previously, to take into account the presence of quantizing magnetic fields. We consider the FCA for the case where the carriers are scattered by acoustic and polar optic phonons. We will present a calculation of the FCA coefficient for electromagnetic radiation polarized along the length of the wire. The magnetic field is assumed to be perpendicular to the wire axis, so that the dispersion of one-dimensional subbands is strongly modified.

2. Formalism

We consider a Q1D electron gas confined in a wire of dimensions L_x, L_y, L_z. We model transverse confinement via an infinite square well approximation to a heterojunction QW (z axis) and a parabolic potential of frequency ω (x axis). Moreover, a magnetic field B, parallel to the z axis, is applied to the wire. The electrons are free in the direction of the wire (y axis). Correspondingly, the one-electron eigenfunctions Ψ_{Nlk_y} and energy eigenvalues

E_{Nlk_y} are given by

$$\Psi_{Nlk_y} = \left(\frac{2}{L_y L_z}\right)^{1/2} \Phi_N(x - x_0) \mathrm{e}^{\mathrm{i}k_y y} \sin\left(\frac{l\pi z}{L_z}\right) \tag{1}$$

$$E_{Nlk_y} = \left(N + \frac{1}{2}\right)\hbar\tilde{\omega}_\mathrm{c} + \frac{\hbar^2 k_y^2}{2\tilde{m}^*} + l^2 E_0 \tag{2}$$

where $N = 0, 1, 2, \ldots$, $l = 1, 2, 3, \ldots$, $E_0 = \pi^2\hbar^2/2m^*L_z^2$, k_y is the wavevector in the y direction, m^* is the effective mass of the electron, $\omega_\mathrm{c} = eH/m^*c$ is the cyclotron frequency, $\tilde{\omega} = \sqrt{\omega_\mathrm{c}^2 + \omega^2}$, and $\tilde{m} = m^*\tilde{\omega}^2/\omega^2$. Moreover, $\Phi_N(x - x_0)$ is the well-known harmonic-oscillator wavefunction centred at $x_0 = \tilde{b}\tilde{R}^2 k_y$ with $\tilde{b} = \omega_\mathrm{c}/\tilde{\omega}$ and $\tilde{R}^2 = \hbar/m^*\tilde{\omega}$.

The FCA coefficient α, which is related to the quantum-mechanical transition probabilities in which the carriers absorb or emit a photon with the simultaneous scattering of the carriers from phonons, is given by [37]

$$\alpha = \frac{\epsilon^{1/2}}{n_0 c} \sum_\mathrm{i} W_\mathrm{i} f_\mathrm{i}. \tag{3}$$

Here ϵ is the dielectric constant of material, n_0 is the number of photons in the radiation field and f_i is the free-carrier distribution function. The sum is over all the possible initial states 'i' of the system. The transition probabilities W_i can be calculated using the standard second-order Born golden rule approximation:

$$W_\mathrm{i} = \frac{2\pi}{\hbar} \sum_{fq} [|\langle \mathrm{f}|M_+|\mathrm{i}\rangle|^2 \delta(E_\mathrm{f} - E_\mathrm{i} - \hbar\Omega - \hbar\omega_q) + |\langle \mathrm{f}|M_-|\mathrm{i}\rangle|^2 \delta(E_\mathrm{f} - E_\mathrm{i} - \hbar\Omega + \hbar\omega_q)]. \tag{4}$$

Here E_i and E_f are the initial and final state energies, respectively, of electrons, $\hbar\Omega$ is the photon energy, $\hbar\omega_q$ is the phonon energy, and $\langle \mathrm{f}|M_\pm|\mathrm{i}\rangle$ are the transition matrix elements from the initial state to the final state for the interaction between electrons, photons and phonons.

The transition matrix elements can be represented by

$$\langle \mathrm{f}|M_\pm|\mathrm{i}\rangle = \sum_\alpha \left(\frac{\langle \mathrm{f}|H_\mathrm{R}|\alpha\rangle\langle\alpha|V_\mathrm{s}|\mathrm{i}\rangle}{E_\mathrm{i} - E_\alpha \mp \hbar\omega_q} + \frac{\langle \mathrm{f}|V_\mathrm{s}|\alpha\rangle\langle\alpha|H_\mathrm{R}|\mathrm{i}\rangle}{E_\mathrm{i} - E_\alpha - \hbar\Omega} \right) \tag{5}$$

where H_R is the interaction Hamiltonian between the electrons and the radiation field, and V_s is the scattering potential due to the electron–phonon interaction.

Using the wavefunctions given by expression (1), the matrix elements of the electron–photon interaction Hamiltonians can be written as

$$\langle k_y' N' l' | H_\mathrm{R} | k_y N l\rangle = -\frac{e\hbar}{m^*} \left(\frac{2\pi\hbar n_0}{V\Omega\epsilon}\right)^{1/2} (\varepsilon\kappa) \delta_{k_y\kappa_y'} \delta_{NN'} \delta_{ll'} \tag{6}$$

where V is the volume of the crystal. Here the radiation field is polarized along the wire, and ε is the polarization vector of the radiation field.

We shall use two different scattering processes: polar-optical scattering and acoustic-phonon scattering. The matrix elements $\langle k_y' N' l' | V_\mathrm{s} | k_y N l\rangle$ of the electron–phonon interaction corresponding to the above two processes are equal to

$$\langle k_y' N' l | V_\mathrm{s} | k_y N l\rangle = C_j' \delta_{k_y', k_y \pm q_y} J_{NN'}(q_x q_y) \Lambda_{ll'}(q_z) \tag{7}$$

where $J_{N',N}(q_x, q_y)$ is the overlap integral of the harmonic wavefunctions:

$$J_{N',N}(q_x, q_y) = \int_{-\infty}^{\infty} \mathrm{d}x \exp(\mathrm{i}q_x x) \Phi_{N'}(x - \tilde{b}\tilde{R}^2 k_y - \tilde{b}\tilde{R}^2 q_y) \Phi_N(x - \tilde{b}\tilde{R}^2 k_y) \tag{8}$$

$$\Lambda_{ll'}(q_z) = \frac{2}{L_z}\int_0^{L_z} \mathrm{d}z \exp(\mathrm{i}q_z z)\sin\left(\frac{l'\pi z}{L_z}\right)\sin\left(\frac{l\pi z}{L_z}\right) \tag{9}$$

$C_j'^2 = C_j^2 F_j(q)$.

The function $\Lambda_{ll'}(q_z)$ given by equation (8) is crucial for our calculation; a suitable approximation is discussed by Ridley [38].

For the electron–polar-optic phonon interaction we have

$$C_{\mathrm{POL}}^2 = 2\pi e^2\hbar\omega_0\epsilon'^{-1}, \qquad F_{\mathrm{POL}} = \frac{N_0^{\pm}}{q^2 V}, \qquad \epsilon'^{-1} = \left\{\frac{1}{\varepsilon_\infty} - \frac{1}{\varepsilon_0}\right\}.$$

Here, ε_∞ and ε_0 are the high-frequency and static dielectric constants of the semiconductor, respectively. As usual, we take the phonon energy $\hbar\omega_q = \hbar\omega_0 \approx$ constant.

$$N_0 = \left[\exp\left(\frac{\hbar\omega_0}{k_{\mathrm{B}}T}\right) - 1\right]^{-1}, \qquad N_0^- = N_0, \qquad N_0^+ = N_0 + 1,$$

where N_0^- (N_0^+) describes the annihilation (creation) of the phonon.

When acoustic phonon scattering is dominant, one may obtain

$$C_{\mathrm{AC}}^2 = \frac{E_{\mathrm{d}}^2 k_{\mathrm{B}}T}{2\rho\upsilon_{\mathrm{s}}^2 V}, \qquad F_{\mathrm{AC}}(q) = 1.$$

In the case of bulk materials and at extremely strong magnetic fields, the electronic wavefunctions have small absolute values of momentum components parallel to the applied magnetic field. Therefore we can neglect the q_z-dependence in the interaction potential given by C_j'.

The electron distribution function for a Q1D nondegenerate electron gas in the presence of a magnetic field can be shown to be

$$f_{Nlk_y} = \frac{2(2\pi)^{1/2}\hbar n_{\mathrm{e}} L_x L_z \sinh(\hbar\tilde{\omega}/2k_{\mathrm{B}}T)}{\delta(\tilde{m}k_{\mathrm{B}}T)^{1/2}} \exp\left\{-\left[\frac{(N+1/2)\hbar\tilde{\omega} + l^2E_0}{k_{\mathrm{B}}T}\right]\right\} \times \exp\left(-\frac{\hbar^2 k_y^2}{2\tilde{m}^* k_{\mathrm{B}}T}\right) \tag{10}$$

where $\delta = \sum_l \exp(l^2E_0/k_{\mathrm{B}}T)$, and n_{e} is the concentration of the electrons.

Below, we will use the following identities:

$$\begin{aligned}
&\int_0^\infty |J_{NN'}(q_x, q_y)|^2 q_\perp \,\mathrm{d}q_\perp = \frac{1}{R^2} \\
&\int_0^\infty |J_{NN'}(q_x, q_y)|^2 q_\perp^3 \,\mathrm{d}q_\perp = \frac{2}{R^4}(N' + N + 1) \\
&\int_0^\infty |\Lambda_{ll'}(q_z)|^2 \,\mathrm{d}q_z = \frac{2\pi}{d}\left(1 + \frac{1}{2}\delta_{ll'}\right).
\end{aligned} \tag{11}$$

Now we make the same approximation as in [4], i.e. we take $\hbar^2/(2\tilde{m}^*)(q_y^2 - 2k_y q_y) = 0$, in δ functions. Using equations (4)–(6) and (9) in (3) and also identities (11), we obtain the following expression for the FCA coefficient for polar and acoustic phonon scattering in a Q1D semiconducting structure in the presence of a magnetic field:

$$\alpha_{\mathrm{POL}}(H) = \frac{4\pi^2 \mathrm{e}^4\hbar\omega_0 n_{\mathrm{e}} \sinh(\hbar\tilde{\omega}/2k_{\mathrm{B}}T)}{c\epsilon^{1/2}\epsilon' m^{*2}\Omega^3 L_z \tilde{b} R^2 \delta} \sum_{N_{\mathrm{f}} l_{\mathrm{f}}} \sum_{N_{\mathrm{i}} l_{\mathrm{i}}} \left(1 + \frac{\delta_{l_{\mathrm{f}} l_{\mathrm{i}}}}{2}\right) \times \exp\left\{-\frac{1}{k_{\mathrm{B}}T}\left[\left(N_{\mathrm{i}} + \frac{1}{2}\right)\hbar\tilde{\omega} + l_{\mathrm{i}}^2 E_0\right]\right\} \times \{N_0\delta((N_{\mathrm{f}} - N_{\mathrm{i}})\hbar\tilde{\omega}$$

$$+ (l_f^2 - l_i^2)E_0 - \hbar\Omega + \hbar\omega_0) + (N_0 + 1)\delta((N_f - N_i)\hbar\tilde{\omega} + (l_f^2 - l_i^2)E_0 - \hbar\Omega - \hbar\omega_0)\} \tag{12}$$

$$\alpha_{AC}(H) = \frac{2\pi e^2 E_d^2 n_e (k_B T)\sinh(\hbar\tilde{\omega}/2k_B T)}{c\rho\epsilon^{1/2}\upsilon_s^2 m^{*2}\Omega^3 L_z \tilde{b}^2 R^4 \delta} \sum_{N_f l_f}\sum_{N_i l_i}\left(1 + \frac{\delta_{l_f l_i}}{2}\right)(N_f + N_i + 1) \times \exp\left\{-\frac{1}{k_B T}\left[\left(N_i + \frac{1}{2}\right)\hbar\tilde{\omega} + l_i^2 E_0\right]\right\} \times \{\delta((N_f - N_i)\hbar\tilde{\omega} + (l_f^2 - l_i^2)E_0 - \hbar\Omega)\}. \tag{13}$$

As can be seen from equations (12) and (13) the FCA coefficient diverges under resonance conditions in a quantizing magnetic field through the δ function. These divergences are associated with the quantization of the electron energy spectrum in the presence of a magnetic field and the confining frequency. This may be removed by replacement of the δ functions by the Lorentzian $\delta_\tau(x) = (\pi\tau)^{-1}/(\tau^{-2} + x^2)$. In this case $\alpha(\omega)$ has δ-function-like spikes with a halfwidth equal to τ^{-1}, whereτ is the phenomenological relaxation time.

It is particularly convenient to express our results in terms of the dimensionless ratio of the FCA coefficient in the presence of the magnetic field to that in the absence of the field. For scattering through acoustic phonons, we adopt the results [30]

$$\alpha_{AC}(0) = \frac{2^{3/2} e^2 E_d^2 (k_B T)^{3/2} n_e \sinh(\hbar\omega/2k_B T)}{m^{*1/2} l_\omega (\hbar\Omega)^3 \epsilon^{1/2}\rho\upsilon_s^2 c L_z \delta} \times \sum_{n_f l_f}\sum_{n_i l_i}\left(1 + \frac{1}{2}\delta_{l_i l_f}\right)\exp\left(-\frac{(n_i + 1/2)\hbar\omega + l^2 E_0}{k_B T}\right) Z\exp(Z)K_1(Z) \tag{14}$$

where

$$Z = \frac{\hbar\Omega - (n_f - n_i)\hbar\omega - (l_f^2 - l_i^2)E_0}{2k_B T},$$

$K_1(x)$ is the modified Bessel function of the second kind, and $l_\omega^2 = \hbar/m^*\omega$. In the quantum limit, in which only the $n_i = n_f = l_i = l_f = 1$ quantum level is occupied and $\hbar\omega_c \gg k_B T$, only the lowest Landau level $N = 0$ is thermally populated; the ratio $\alpha_{AC}(H)/\alpha_{AC}(0)$ and $\alpha_{POL}(H)/\alpha_{POL}(0)$ are functions of $\tilde{\omega}T$, Ω. It can be seen that the ratio depends only upon the magnetic field, absolute temperature, and photon frequency and does not depend upon such material parameters as the values of the deformation potential, sound velocity, or density of the material, although, of course, the absolute value of absorption coefficient does depend upon the numerical values of these parameters.

3. Discussion

Thus, we have obtained general expressions for FCA coefficients for QWWs in the presence of the quantizing magnetic field. From equations (12) and (13) it can be seen that, in the extreme quantum limit ($\hbar\tilde{\omega} \gg k_B T$, $N_i = 0$, $l_i = l_f = 1$) for polar optical phonons, the FCA coefficient oscillates as a function of the magnetic field and photon frequency with resonances occurring when $P\tilde{\omega} = \Omega \pm \omega_0$. Since $\omega_c < \tilde{\omega}$, for $\omega > 0$, the resonances are shifted to smaller magnetic fields. The above conditions give the resonance magnetic fields H as $H = \sqrt{(\Omega \pm \omega_0)^2(m^*c/e)^2 - (cm^*\omega/e)^2}/P$. For $\omega = 0$, i.e., in the absence of confinement, $\tilde{b}^2 = 1$, $\tilde{\omega} = \omega_c$, and we recover the usual resonance condition $P\omega_c = \Omega \pm \omega_0$. For elastic scattering by acoustic phonons, resonances are expected when $P\tilde{\omega} = \Omega_0$.

We will consider that L_z is so small that no transitions between levels l can take place due to thermal excitations or phonons. For GaAs, E_0 is about 0.05 eV for $d = 100$ Å, and

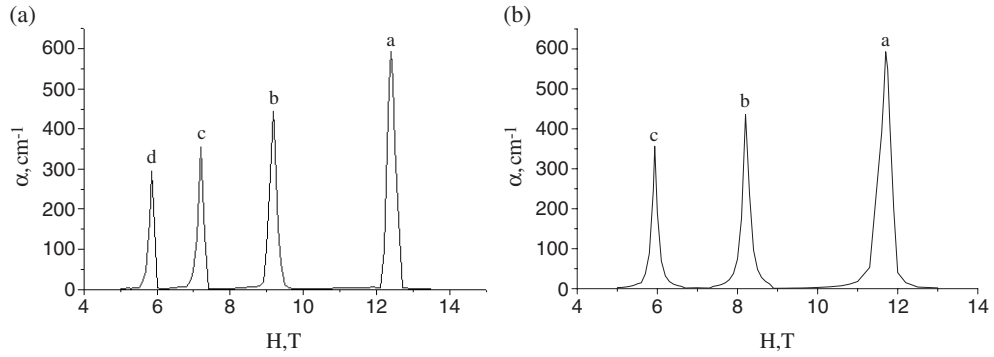

Figure 1. (a) For $\hbar\omega = 0.004$ eV, we present the variation of the FCA coefficient α_{POL} as a function of the magnetic field at $\hbar\Omega = 0.1$ eV; (b) for $\hbar\omega = 0.008$ eV, we plot the absorption coefficient α_{POL} as a function of the magnetic field at $\hbar\Omega = 0.1$ eV.

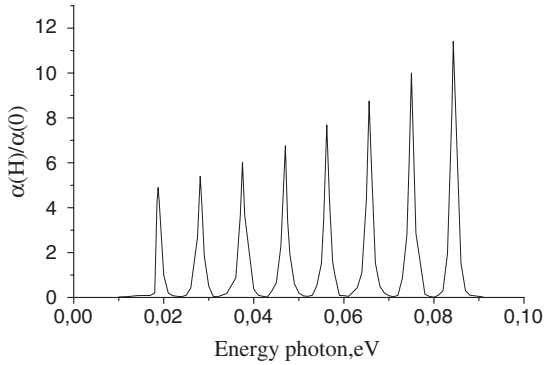

Figure 2. The ratio of the FCA coefficient in the presence of a magnetic field to its zero-field value is shown as a function of photon frequency for acoustic phonons.

$\hbar\omega_{\mathrm{c}} = 1.7H$ (meV), with H measured in tesla units. That is, we consider that all the carriers are in the lowest subband $l_{\mathrm{i}} = l_{\mathrm{f}} = 1$. We have evaluated the FCA coefficients numerically, in the extreme quantum limit, for GaAs. The parameters used in our calculation are $\tau = 10^{-12}$ s, $T = 100$ K, $m^* = 0.07m_0$, $\hbar\omega_0 = 0.036$ eV.

In figure 1(a), for $\hbar\omega = 0.004$ eV, we present the variation of the FCA coefficient α_{POL} as a function of the magnetic field at $\hbar\Omega = 0.1$ eV. The resonances in α versus H are noted. It is shown that the amplitude of the oscillation increases with the magnetic field. Since we introduced a broadening to the δ function, the divergence at the resonance is removed. The oscillatory dependence of the absorption on the magnetic field can be understood in terms of the Landau subband structure of the electronic energy levels in quantizing magnetic fields. As the magnetic field, and therefore $\tilde{\omega}$, increases there are fewer and fewer subbands to which the transition can occur. Every time that the ratio $(\Omega \pm \omega_0)/\tilde{\omega}$ equals an integer value, the transition can take place with an additional subband ending as a final state.

In figure 1(b), for $\hbar\omega = 0.008$ eV, we plot the absorption coefficient α_{POL} as a function of the magnetic field at $\hbar\Omega = 0.1$ eV. By comparing figures 1(a) with (b) we see that the effect of confinement in the x direction for the Q1D quantum wire structure is to shift the ordinary resonance peak position to lower magnetic field.

The ratio of the FCA coefficient in the presence of a magnetic field to its zero-field value is shown as a function of photon frequency, at $H = 5$ T, for acoustic phonons in figure 2. As with the magnetic field dependence, the resonances in α versus $\hbar\Omega$ are noted.

In conclusion, we predict that the FCA coefficient should increase with magnetic field with an oscillatory dependence on the field when $\Omega > \tilde{\omega}$. The magnetic field dependence of the FCA coefficient is explained in terms of the field dependence of the scattering rates and the possibility of phonon-assisted transitions between various Landau levels when $\Omega > \tilde{\omega}$. The effect of confinement in the x direction for the Q1D quantum wire structure is to shift the ordinary resonance peak position to lower magnetic field.

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